

A comment on “Exclusion of the remaining mass window for primordial black holes...”, arXiv:1401.3025

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In the recent paper [1], A. Loeb and P. Pani have proposed a new energy loss mechanism by primordial black holes (PBH) of the mass $10^{17} - 10^{26}$ g passing through a neutron star (NS). This mechanism is claimed to be many orders of magnitude more efficient than the dynamical friction mechanism that was used in the previous estimates [2]. As a result, the constraints on the abundance of PBH were improved by many orders of magnitude. In this comment we point out a potential problem in the calculations of Ref. [1] that may invalidate their result. In that case, the new mechanism would give parametrically the same energy loss as the dynamical friction, and would not imply a significant improvement of the constraints on PBHs.

The difference between the two calculations of the energy losses is that in Ref. [2] the NS matter is treated as approximately pressureless during its interaction with the PBH. In this approach, the energy losses result from gravitational scattering of individual neutrons on the passing PBH. On the contrary, in Ref. [1] the NS matter is treated as a gravitating fluid, and the energy losses result from excitation of acoustic oscillations of this fluid by a moving PBH.

Despite apparent difference in the approaches, the huge mismatch in the results is difficult to believe because of causality reasons. A PBH falling on a NS attains a relativistic velocity $\sim 0.6c$ (in the middle of the star). This velocity is larger (in the center) or much larger (in the outer core and in the crust) than the speed of sound. When the NS matter gets perturbed by the passing PBH, there is no time for it to organize itself in any kind of acoustic waves because of the slow sound speed. In other words, at the time scale of the interaction with the PBH the NS matter behaves approximately as pressureless. In this approximation, the answer for the energy loss has been calculated by Chandrasekhar [3], the corresponding mechanism being known as the dynamical friction. Within this picture, the alternative approach in terms of the sound waves may be viewed simply as a different way of assessing the energy losses in which the perturbation created by the BH, with the energy losses already encoded in it, is first represented as a sum of acoustic modes, and then energies of the individual modes are

summed up. Thus, in the limit when the PBH velocity is much larger than the speed of sound, the answer should be the same whatever approach is used.

The fact that the pressureless approximation is accurate in the case of a supersonic motion has been confirmed by an explicit calculation in Ref. [4], where it was shown that already for a PBH passing through a homogeneous medium with velocity larger than 2 – 3 times the sound speed, the energy loss to the emission of sound waves reproduces exactly the dynamical friction formula. Thus, no large differences between the acoustic oscillation approach and the dynamical friction one should be expected in the case of a PBH crossing a NS.

With this understanding in mind, consider how the large enhancement factor of Ref. [1] comes about. The approach of Ref. [1] is based on the calculation of the excitations of seismic modes of a NS by a passing PBH. The dominant contribution was argued to come from the fundamental $n = 0$ modes, the so-called f -modes that are, loosely speaking, surface waves at the surface of the NS core. These waves are supported by the combination of gravity and pressure, not the pressure alone as usual sound waves, hence a possible difference with the calculation of Ref. [4]. Note however, that these waves still do not propagate faster than the speed of sound, so the dynamical friction result is still expected in the supersonic case. The contributions of these f -modes to the total energy loss were calculated numerically up to $l = 100$ and found to follow the behavior

$$E_l \propto \frac{m_{\text{BH}}^2}{R} \frac{1}{l^n}, \quad (1)$$

where m_{BH} is the BH mass, $R = 10\text{km}$ is the NS radius and $n \sim 0.5$ is related to the equation of state (EOS) in the NS core. The coefficient m_{BH}^2/R is, up to a numerical factor, the energy loss due to the dynamical friction [2]. So, the sum $\sum_l l^{-n}$ represents the enhancement compared to the dynamical friction result (again, up to a numerical factor). The sum is divergent. The authors of Ref. [1] cut the sum at the BH size which corresponds to $l_{\text{max}} \sim RM_{\text{Pl}}^2/m_{\text{BH}} \sim 10^{10}$ for $m_{\text{BH}} = 10^{24}$ g. Extrapolating eq. (1) from $l = 100$ to $l = 10^{10}$ and evaluating the sum then gives the enhancement factor $l_{\text{max}}^{1-n} \sim 10^5 \gg 1$.

We think that the discrepancy between this result and the dynamical friction calculation may be explained by the breaking of eq. (1) beyond l of order several hundred. The physical reason for this is that the core-crust transition in a NS — the region relevant for deriving eq. (1) — has a finite thickness (cf. [7] where a similar idea is discussed in the context of Solar oscillations). The characteristic distance scale can be estimated as $d = \rho/(d\rho/dr)$. From the realistic NS density profile [5] we have obtained $d \sim 30$ m, which corresponds to $l_d \sim 500$. At the wavelengths much smaller than d (equivalently, for $l \gg l_d$) the character of the f -modes must change and eq. (1) should get modified. Note that eq. (1) has to be extrapolated to much larger values $l \sim 10^{10}$ in order to get the claimed enhancement factor.

In mathematical terms, the break in E_l can be understood as follows. Equation for the acoustic perturbations in the gravitating fluid, in the Cowling approximation and in the limit of vanishing buoyancy frequency (which is identically zero for a polytropic EOS), can be written as [6]

$$\xi'' + \left(\frac{4}{r} + \frac{\rho'}{\rho}\right)\xi' + \left(\frac{\rho'}{r\rho} - \frac{l(l+1)-2}{r^2}\right)\xi = -\frac{\omega^2}{v_s^2}\xi, \quad (2)$$

where the prime denotes the derivative with respect to r , ξ is the angular displacement, v_s is the sound speed and ρ is the density. Note the appearance of the combination ρ'/ρ , the inverse distance scale. To understand the behavior of solutions one may eliminate the first derivative by an appropriate change of variables $\xi = f(r)\eta$. This gives

$$\eta'' + \left(-\frac{\rho''}{2\rho} - \frac{\rho'}{r\rho} + \frac{1}{4}\frac{\rho'^2}{\rho^2} - \frac{l(l+1)}{r^2}\right)\eta = -\frac{\omega^2}{v_s^2}\eta. \quad (3)$$

Since $\rho'/\rho = -g/v_s^2$, g being the gravitational acceleration, the ρ -dependent terms grow toward the surface where the sound speed becomes small. For a polytropic EOS, neglecting the variation of g , one has $\rho'/\rho \propto -(R-r)^{-1}$, R being the boundary of the star. As one can see from eq. (3), in the region away from the surface and for large l , the term $l(l+1)/r^2$ dominates and there are no solutions except for normal sound waves with the dispersion relation $\omega^2 = v_s^2(k_r^2 + k_{||}^2)$, where k_r and $k_{||}$ are the radial and angular components of the momentum. The surface waves, for which $k_r \sim 0$ and the dispersion relation is linear in $k_{||}$, have no support in this

region. They are concentrated in the layer of the thickness $\sim R/l$ next to the boundary where the $l(l+1)/r^2$ term can be balanced by other contributions. This is a known result [6].

In a real NS, the perturbation equations are more complicated [8]. Still, we think that eq. (2) grasps the main features and is sufficient for a rough estimate. For a realistic NS density profile, the ρ -dependent terms in eq. (3) grow with increasing r and reach the value of $\sim 1/d^2$ around the core-crust transition region, as set by the finite thickness $\sim d$ of this region, and then grow to much larger values toward the surface of the star. Thus, for $l \lesssim l_d \sim R/d$, eq. (3) allows for the existence of the f -modes which peak in the core-crust transition region and may produce eq. (1). However, at $l \gtrsim l_d$ the term $l(l+1)/r^2$ in eq. (3) becomes dominant in this region. Therefore, the f -modes with such l peak in the crust and are exponentially small around the core-crust transition. For this reason they know nothing about the EOS in the core, and thus the corresponding E_l cannot follow eq. (1) with $n \simeq 0.5$ characteristic of the core. The behavior of E_l must, therefore, change.

To summarize, the extrapolation of eq. (1) from $l = 100$ to $l \sim 10^{10}$ (for $m_{\text{BH}} = 10^{24}$ g) is unjustified. Moreover, one should expect a suppression in eq. (1) at l of order several hundred. If only modes up to $l \sim 100$ in eq. (1) are summed up, the answer is not different from the dynamical friction result apart from possibly a numerical factor of order 1, in accord with the causality arguments.

The expected break in E_l may not be the only problem of the calculation of Ref. [1], the other one being the justification of the linear approximation. The justification given in Ref. [1] — that the energy losses are much smaller than the rest mass of the PBH — does not sound convincing, because it is not clear how does this fact translate into smallness of the non-linear terms in the full hydrodynamic equations — the terms that have been omitted when deriving the linearized mode equations for perturbations. Certainly, a more detailed study of this issue is required.

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- [1] P. Pani and A. Loeb, arXiv:1401.3025 [astro-ph.CO].
 - [2] F. Capela, M. Pshirkov and P. Tinyakov, Phys. Rev. D **87** (2013) 123524 [arXiv:1301.4984 [astro-ph.CO]].
 - [3] Chandrasekhar, S. 1943, Astrophys. J. , 97, 255
 - [4] Ostriker, E. C. 1999, Astrophys. J. , 513, 252
 - [5] Belvedere, R., Pugliese, D., Rueda, J. A., Ruffini, R., & Xue, S.-S. 2012, Nuclear Physics A, 883, 1
 - [6] Smeyers, P. 2010, Linear Isentropic Oscillations of Stars:

- Theoretical Foundations, Astrophysics and Space Science Library, Volume 371. ISBN 978-3-642-13029-8. Springer-Verlag Berlin Heidelberg, 2010.
- [7] Rosenthal, C. S., & Christensen-Dalsgaard, J. 1995, Mon. Not. R. Astron. Soc., 276, 1003
- [8] J. Ruoff, Phys. Rev. D **63** (2001) 064018 [gr-qc/0003088].